An Air Brake Hybrid Model for Trains

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Motivation: towards automatic train operation

Besides the obvious advantages of automatic driving for vehicles, trains include additional characteristics:

- Wide performance variation among operators
- 3-5 years experience to qualify to drive
- Aging workforce
- Increased rail demand

Control of tractive and braking effort is a main problem in ATO. For all this, a faithful model is needed for the different components of a train.

Trains are typically equipped with different braking systems, depending on the characteristics of the trains, loads, operating speed and track conditions, among others. In this talk we focus on air brake because of its challenging behaviour, that needs to be modelled as a hybrid system.

A symplified model of a train with air brakes

Train model –3 states, nonlinear ODEs, nonsmooth

$$\dot{\xi}(t) = v(t)$$

$$\dot{v}(t) = \frac{1}{M}\left(f(p(t),v(t)) - \mu(v(t)) - g\sin(\theta(\xi)) - b(t)\right)$$

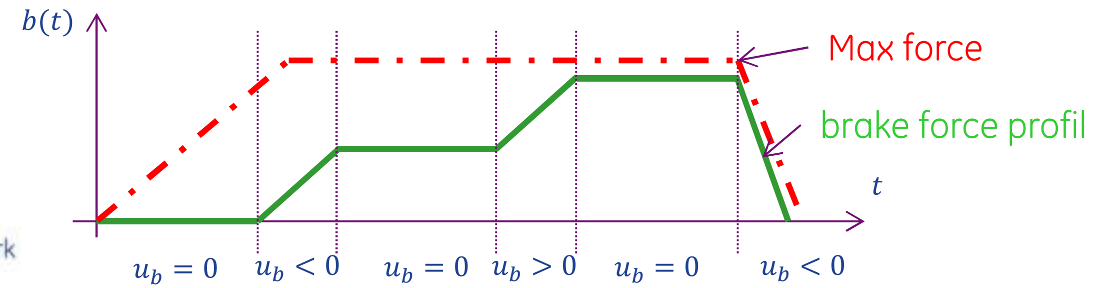
engine effort: $f(p(t),v(t))$; propulsion resistance: $\mu(v(t))$; gravity: $g\sin(\theta(\xi))$; air Brake effort: $b(t)$

$$f(t) = \begin{cases} \frac{p(t)}{\bar{V}} & if \quad v(t) < \bar{V} \\ \frac{p(t)}{v(t)} & otherwise \end{cases}$$

$$\dot{p}(t) = \begin{cases} \alpha u_{d(t)} & if \quad \underline{P} < p(t) < \bar{P} \\ 0 & otherwise \end{cases}$$

Air Brake model – hybrid system

There are 3 discrete states: idle, brake and release, each with different dynamics. The maximum force provided by this brake is also a function of past behaviours.



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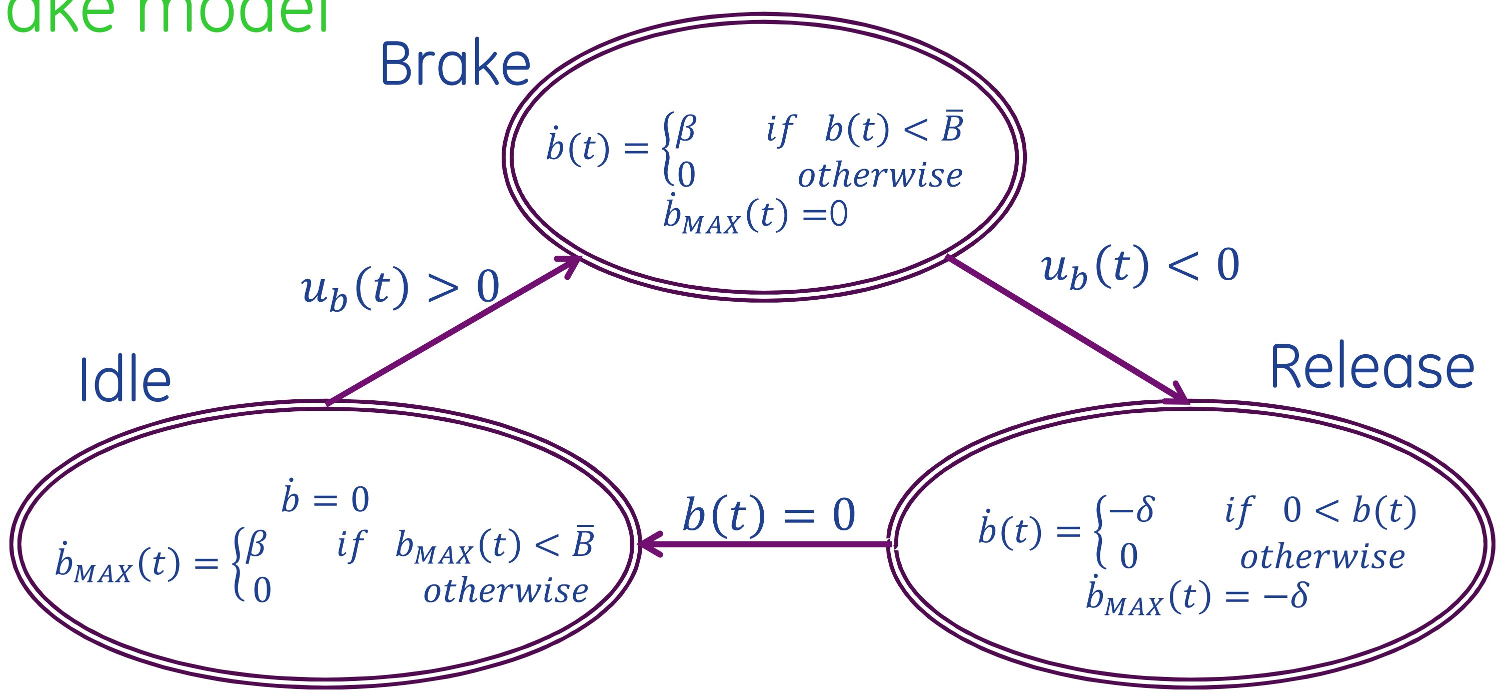
State machine / Hybrid formulation

Air Brake model



Combined model

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$$\frac{d}{dt}\begin{pmatrix} x \\ \zeta \end{pmatrix} = \begin{pmatrix} F(x,u) \\ 0 \end{pmatrix}, \qquad q \in C$$

$$\begin{pmatrix} x^+ \\ \zeta^+ \end{pmatrix} = \begin{pmatrix} x \\ \zeta + 1 \; (mod \; 3) \end{pmatrix}, q \in D$$

$$u = (u_d, u_b), x = (\xi, v, p, b, b_{MAX}), q = (x, \zeta)$$
$$C \subseteq \mathbb{R}^5, D = D_1 \cup D_2 \cup D_3$$
$$D_1 = \{q: \zeta = 1, u_b \geq 0\}, D_2 = \{q: \zeta = 2, u_b \leq 0\},$$
$$D_3 = \{q: \zeta = 3, b = 0\}$$

Control Problem / Requirements

There are two control inputs, the engine force (tractive or braking) and the air brake force. The goal of the strategy is to track a desired, pre computed optimal speed profile while minimizing energy consumption.

$$J(u, q_0) = \int_0^\infty u(\tau)^T R u(\tau) d\tau + \int_0^\infty \left(v(\rho; u, q_0) - \bar{v}(\rho)\right)^2 d\rho$$

This optimization is subject to a safety constraint, namely, that the speed should be always under the maximum allowed speed

$$\min_{|u|_\infty \leq 1} J(u, q_0)$$

$$\text{s.t. } v(\xi; u, q_0) \leq \bar{V}(\xi) \;\; \& \;\; ①$$

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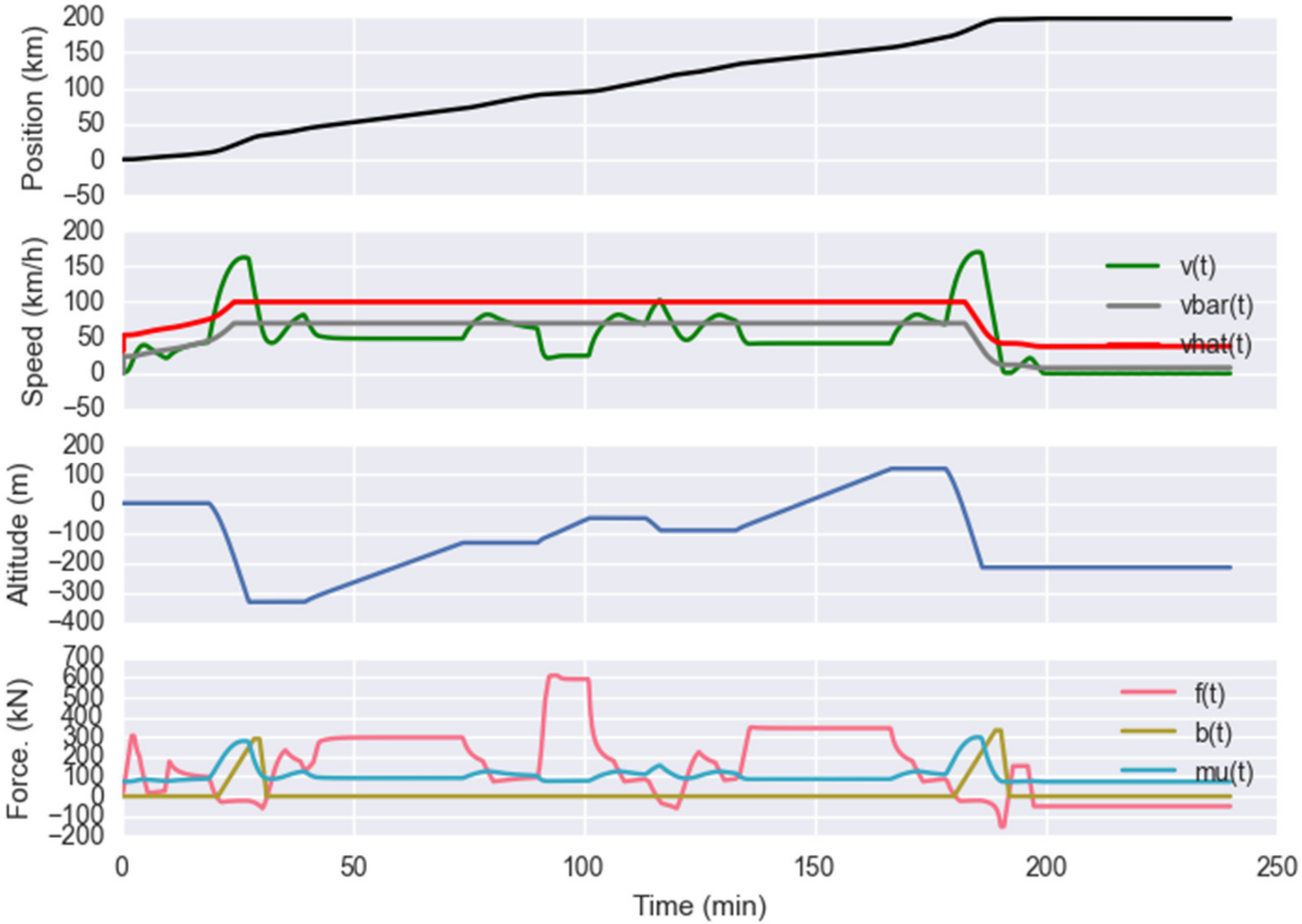
Implementation / Example

A possible implementation of the proposed model has been developed in Python. It includes a simple controller. Below is a typical run:



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Possible Extensions

Several possible extensions can make the model more realistic:

- The train is modelled here as a point-mass, instead of a more realistic distributed mass model.

- With increasing production of natural gas, duel fuel engines become more attractive for trains as well. When a dual fuel engine is modelled correctly, also the train model itself becomes a hybrid system.

- Uncertainties of up to 50% of the nominal value of the air braking force are possible.

- Delays in brake actuation are far from being negligible.

- Air brakes cannot be used continuously for a long time, since they heat up and lose efficiency.

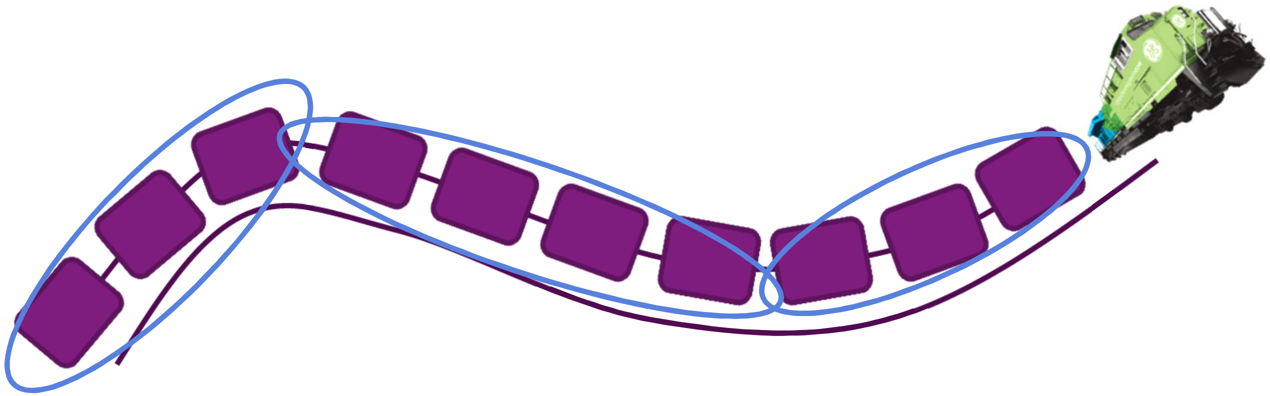
- In terms of requirements, one can also impose a constraint in the maximum forces that appear between train cars (to avoid bunching & stretching).

- Human factors should be considered as well – how would a driver react to an optimal profile?

- Coarse quantization for all control inputs.

Possible Extensions

- Distributed control is possible when several locos are present, or under electronically controlled pneumatic brakes – how to lump inputs in groups?



$$\frac{dx}{dt} = F(x,u), x \in \mathbb{R}^n, u \in \mathbb{R}^p \;\rightarrow\; \frac{dx}{dt} = F(x,u), x \in \mathbb{R}^n, u \in \mathbb{R}^q, \qquad q \ll p$$

This lumping procedure should vary over position, depending on the gradient

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